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Supergravity solution of intersecting branes and AdS/CFT with flavor

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Abstract: We construct the supergravity solution for fully localized D2/D6 intersection. The near horizon limit of this solution is the supergravity dual of supersymmetric Yang-Mills theory in 2+1 dimensions with flavor. We use this solution to formulate mirror symmetry of 2+1 dimensional gauge theories in the language of AdS/CFT correspondence. We also construct the supergravity dual of a non-commutative gauge theory with fundamental matter.

Keywords: Duality in Gauge Field Theories, AdS-CFT and dS-CFT Correspondence.

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1. Introduction

Supergravity solutions of intersecting D-branes are relatively easy to find as long as they are sufficiently smeared [1]. Supergravity solutions of the localized intersections are far more difficult to find. Starting with the work of [2] there has been a steady enterprise of attempts to construct such supergravity solutions [3]–[20]. However, to date, there are no known techniques for determining the gravitational back reaction due to a general intersecting brane configurations that arise in string theory.

For brane intersections involving a D6-brane, there are special techniques which allow the explicit construction of certain localized intersections [5, 6, 7]. Here, one takes advantage of the fact that the M-theory lift of the geometry of the D6-brane near its core is an ALE space which is essentially the flat $\mathbb{R}^4$. This allowed the construction of the completely localized supergravity solution of D2 parallel to D6, as well as D4 ending on the D6. This method however was limited in its applicability to the region near the core of the D6.

The aim of this paper is to construct the fully localized supergravity solution of D2 parallel to D6 without restricting to the near core region of the D6. The construction of the supergravity solution turns out to be possible for this case due to the fact that there exists a simple ansatz which reduces the problem to a single linear differential equation [2, 3] which is separable [8] and admits a regular boundary condition [9]. Therefore, the problem can be solved using elementary methods. The solutions we obtain are nonetheless very interesting. Instead of taking the near horizon limit of the D6-brane as was done in [5, 9], one can consider the near horizon limit of the D2-brane. This gives rise to a geometry where a D6-brane is slicing through the near horizon geometry of the D2-branes. From the open string point of view, this corresponds to taking the decoupling limit which keeps only the gauge fields on the D2-brane and the charged fundamental matter arising from strings stretching between the D2 and the D6 branes. It is therefore the 2+1 dimensional version of the holographic dual of gauge theory with flavors considered recently by Katz and Karch in [21] for the 3+1 dimensional case. There are several advantages for considering the case of 2+1 dimensions over 3+1. The field theory in 2+1 dimensions is superrenormalizable even after adding the fundamental matter, in contrast to 3+1 dimensional theory which looses the asymptotic freedom.[1] The supergravity solution takes the full gravitational back reaction of the D6-branes into account in contrast to the 3+1 example where the effect of D7-brane is treated in the probe approximation. One can therefore think of the localized D2/D6 solution as the "cleaner" version of the AdS/CFT-like correspondence with fundamental flavors.

The near horizon limit of the D2/D6 solution captures the full RG flow of the weakly coupled supersymmetric Yang-Mills theory with fundamental matter in the UV to a superconformal fixed point in the IR along the lines of [24]. In fact, certain qualitative features of precisely this RG flow was anticipated in [25]. Our explicit supergravity solution confirms the expectation of [25]. As an added bonus, these supergravity solutions can be used to illustrate the mirror symmetry of Intriligator and Seiberg [26] in the language of AdS/CFT correspondence.

2. The solution

Let us begin by describing the explicit construction of the supergravity solution. The idea is to start with a lift of D6-brane to the Taub-NUT geometry in M-theory and to consider the effect of placing large number of M2-branes in this background. To describe this background, one employs the ansatz

$$\begin{aligned} ds^2 &= H^{-2/3}(-dt^2 + dx_1^2 + dx_2^2) + H^{1/3}(dy^2 + y^2 d\Omega_3^2 + ds_{TN}^2) \\ F &= dt \wedge dx_1 \wedge dx_2 \wedge dH^{-1} \end{aligned} \tag{2.1}$$

where ds_{TN}^2 is the metric of the Taub-NUT space. We find it convenient to parameterize the coordinates of the Taub-NUT space so that the metric takes the form

$$ds_{TN}^2 = \left(1 + \frac{2m}{r}\right)(dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)) + \left(\frac{1}{1+2m/r}\right)(4m)^2\left(d\psi + \frac{1}{2}\cos\theta d\phi\right)^2 \tag{2.2}$$

where the coordinate take on values with range $0 \leq r$, $0 \leq \theta \leq \pi$, $0 \leq \phi \leq 2\pi$, and $0 \leq \psi \leq 2\pi$. The parameter m is related to the radius R of the circle of the Taub-NUT metric at infinity by the formula

$$R = 4m\,. \tag{2.3}$$

For small radius

$$r = \frac{z^2}{8m} \ll m\,, \tag{2.4}$$

[1] There are, however, constructions involving orientifolds which maintain conformal invariance [22, 23].

the Taub-NUT metric simplifies to

$$ds_{TN}^2 = dz^2 + z^2 d\Omega_3^2 \,. \tag{2.5}$$

The ansatz (2.1) is a solution to the equation of motion of 11 dimensional supergravity if H solves the harmonic equation in the background of $\mathbb{R}^4 \times$ Taub-NUT space

$$\nabla^2 H = 0, \tag{2.6}$$

except at the location of the M2-brane source. In order to maximize the symmetry of the problem to simplify the analysis, let us consider the case where the M2-brane source is placed at the origin $y = r = 0$. One can than take $H(y, r)$ to be a function of two variables, and the harmonic equation (2.6) becomes

$$\left(\frac{1}{1+2m/r}\right)\left(\frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r}\right) H(y,r) + \nabla_y^2 H(y,r) = 0 \,. \tag{2.7}$$

Our task is simply to solve this differential equation. To this end, it is convenient to separate variables

$$H(y,r) = 1 + Q_{M2}\int \frac{d^4p}{(2\pi)^4} e^{ipy} H_p(r) \,, \tag{2.8}$$

where Q_{M2} is the membrane charge in the standard normalization

$$Q_{M2} = 32\pi^2 N_2 l_p^6 \,. \tag{2.9}$$

Then, H_p satisfies

$$\left(\frac{1}{1+2m/r}\right)\left(\frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r}\right) H_p(r) - p^2 H_p(r) = 0 \,. \tag{2.10}$$

Since this is a second order differential equation, there are two solutions. The one which decays at large r can be written in a closed analytic form

$$H_p(r) = c_p e^{-pr}\mathcal{U}(1 + pm, 2, 2pr) \,, \tag{2.11}$$

where $\mathcal{U}(a, b, z)$ is the confluent hypergeometric function [27]. The normalization factor c_p is fixed to

$$c_p = \frac{\pi^2}{8}\frac{1}{m^2}(pm)^2\Gamma(pm) \tag{2.12}$$

by requiring that in the $m \to \infty$ limit keeping $z^2 = 8mr$ fixed (which is equivalent to looking at $r \ll m$), $H_p(r)$ becomes (using 13.3.3 of [27])

$$H_p(z) = \frac{\pi^2}{2z^2}\, pz K_1(pz) \tag{2.13}$$

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whose Fourier transform is

$$\int \frac{d^4p}{(2\pi)^4} e^{ipy} H_p(z) = \frac{1}{(y^2+z^2)^3} \,. \tag{2.14}$$

We finally arrive at the statement

$$H(y,r) = 1 + Q_{M2} \int dp \, \frac{(py)^2 J_1(py)}{4\pi^2 y^3} H_p(r) \tag{2.15}$$

where we have reduced (2.8) to an integral over a single variable by exploiting the spherical symmetry in p-space.

The solution (2.15) combined with the ansatz (2.1) is the main result of this paper. Dimensional reduction of this solution along the ψ coordinate of the Taub-NUT geometry (2.2) will give rise to the solution type IIA supergravity describing D2 localized along the world volume of D6. The metric part of the solution is given by

$$\begin{aligned} ds^2 &= H(y,r)^{-1/2}\left(1+\frac{2m}{r}\right)^{-1/2}(-dt^2+dx_1^2+dx_2^2)+ \\ &+H(y,r)^{1/2}\left(1+\frac{2m}{r}\right)^{-1/2}(dy^2+y^2d\Omega_3^2)+H(y,r)^{1/2}\left(1+\frac{2m}{r}\right)^{1/2}(dr^2+r^2d\Omega_2^2)\,, \end{aligned} \tag{2.16}$$

where $4m = R = g_s l_s$. It is clear that when we set $Q_{M2}=0$, the solution reduces to the supergravity solution containing only the D6-branes. Similarly, in the $m \to 0$ limit,

$$H(y,r) = 1 + \frac{Q_{D2}}{(y^2+r^2)^{5/2}}\,, \tag{2.17}$$

where using $l_p = g_s^{1/3} l_s$,

$$Q_{D2} = \frac{3}{64m} Q_{M2} = 6\pi^2 g_s N_2 l_s^5\,, \tag{2.18}$$

which agrees with the supergravity solution of the D2 by itself including the numerical factors. Although (2.15) is left in an integral form, the expression is completely explicit and the final integration can be done numerically if desired. To demonstrate this point, we have computed

$$f(r) = \frac{512 m^3 r^3}{Q_{M2}}(H(0,r)-1) \tag{2.19}$$

numerically. The normalization was chosen so that $f(r)=1$ for $r \to 0$. The result of this computation is illustrated in figure 1. The result clearly illustrates the cross-over between asymptotics (2.14) and (2.17) for small and large r, respectively.

3. The decoupling limit

Now that we have constructed the supergravity solution of the localized intersection of D2 and D6 in type IIA supergravity, let us consider taking its near horizon limit which gives rise to a holographic dual of the gauge theory on the D2-branes. We will scale l_s to zero keeping the two dimensional gauge coupling

$$g^2_{\mathrm{YM2}} = g_s l_s^{-1} \tag{3.1}$$

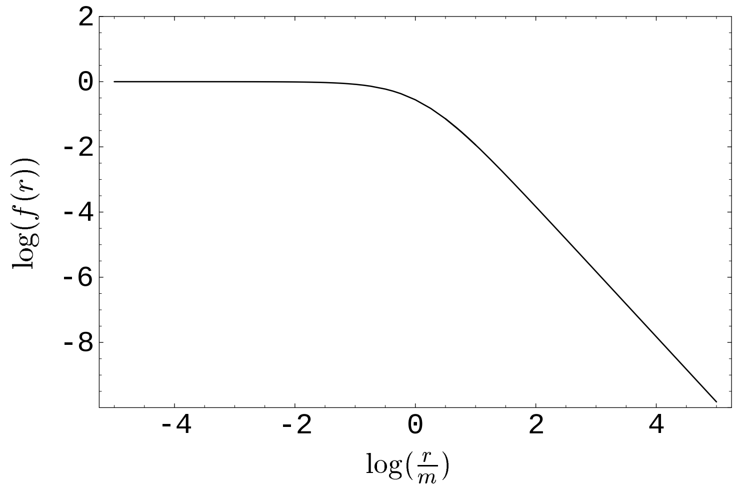

Figure 1: Log-Log plot of the function $f(r)$.

fixed. In this limit, the gauge coupling on the six brane

$$g^2_{\mathrm{YM6}} = (2\pi)^4 g_s l_s^3 = (2\pi l_s)^4 g^2_{\mathrm{YM2}} \tag{3.2}$$

goes to zero so that the dynamics on the 6-brane decouples. In order to identify the corresponding near horizon geometry on the supergravity side, we also scale the radial coordinates so that

$$Y = \frac{y}{l_s^2}, \qquad U = \frac{r}{l_s^2} \tag{3.3}$$

is fixed. In this limit, the harmonic function due to the D6-brane source scales as

$$1 + \frac{2m}{r} = 1 + \frac{g_s l_s}{2r} = 1 + \frac{g^2_{\mathrm{YM2}}}{2U}, \tag{3.4}$$

whereas the harmonic function due to the D2-brane source (2.15) scales as

$$H(Y,U) = \frac{1}{l_s^4} h(Y,U), \tag{3.5}$$

where

$$h(Y,U) = \pi^2 g^4_{\mathrm{YM2}} N_2 \int dP \frac{(PY)^2 J_1(PY)}{Y^3} P^2 \Gamma\left(\frac{g^2_{\mathrm{YM2}} P}{4}\right) e^{-PU} \mathcal{U}\left(1 + \frac{g^2_{\mathrm{YM2}} P}{4}, 2, 2PU\right). \tag{3.6}$$

We have also scaled the integration variable

$$p = \frac{P}{l_s^2} \tag{3.7}$$

so that the string length l_s does not appear anywhere in the definition of $h(Y,U)$.

We now have all the ingredients to explicitly write down the supergravity solution for the decoupled D2/D6 system. Generalizing slightly to the case with multiple coincident

D6-branes, the solution takes the form

$$\begin{aligned}
\frac{ds^2}{l_s^2} &= h(Y,U)^{-1/2}\left(1+\frac{g_{\rm YM2}^2 N_6}{2U}\right)^{-1/2}(-dt^2+dx_1^2+dx_2^2)+\\
&\quad + h(Y,U)^{1/2}\left(1+\frac{g_{\rm YM2}^2 N_6}{2U}\right)^{-1/2}(dY^2+Y^2 d\Omega_3^2)+\\
&\quad + h(Y,U)^{1/2}\left(1+\frac{g_{\rm YM2}^2 N_6}{2U}\right)^{1/2}(dU^2+U^2 d\Omega_2^2) \qquad (3.8)\\
h(Y,U) &= \pi^2 g_{\rm YM2}^4 N_2 \int dP \frac{(PY)^2 J_1(PY)}{Y^3} P^2 \Gamma\left(\frac{g_{\rm YM2}^2 N_6 P}{4}\right)\times\\
&\quad \times e^{-PU}\mathcal{U}\left(1+\frac{g_{\rm YM2}^2 N_6 P}{4}, 2, 2PU\right). \qquad (3.9)
\end{aligned}$$

The only dependence of this metric on the string length l_s is in the overall normalization which is what one expects for the supergravity dual of a quantum field theory. Note also that although we no longer have the "1" in the harmonic function of the D2-brane in taking the decoupling limit, we are left with the "1" in the harmonic function of the D6-brane. The effect of the D2-brane is therefore to "warp" not only the ALE region but also the asymptotically flat region of the D6-brane geometry.

4. RG flow and mirror symmetry

Let us now interpret the various features of the supergravity solution (3.8) from the point of view of the field theory dual. By construction, (3.8) is dual to maximally supersymmetric SYM in 2+1 dimensions with gauge group SU(N_2) further coupled to $k = N_6$ flavors of massless hypermultiplets in the fundamental representation. The coupling to the hypermultiplets reduces the number of unbroken supersymmetries from 16 to 8. The supergravity solution (3.8) asymptotes to the geometry of the near horizon D2-brane in the large U limit. This suggests that the dynamics of this theory is dominated by the free gluons in the UV as one expects for a superrenormalizable theory. In the small U region, the geometry of (3.8) asymptotes to $AdS_4 \times S_7/\mathbb{Z}_k$. This is the superconformal field theory one expects to find on M2-brane probing $\mathbb{R}^4 \times (\mathbb{R}^4/\mathbb{Z}_k)$.

Now, the superconformal theory on the M2-brane on an orbifold does not have a simple lagrangian formulation. One way to define such a theory without relying on string theory is to define it as an IR fixed point of a different theory which has a lagrangian formulation. The 2+1 SYM with k flavors is one concrete example of a UV theory which flows to this superconformal field theory in the IR. Roughly speaking, by compactifying one of the directions of $\mathbb{R}^4/\mathbb{Z}_k$ into a Taub-NUT space, one has embedded the dynamics of M2 into the dynamics of D2, and it is the latter which has a good lagrangian description. By making the D2 warp the $\mathbb{R}^4 \times$ Taub-NUT geometry due to its gravitational backreaction, one constructs the supergravity dual to the decoupled theory on the D2-brane. The supergravity solution (3.8) encodes this full renormalization group flow in the language of AdS/CFT correspondence. Similar observations can be found in the earlier work of [25].

It turns out that there is a different way to embed the superconformal field theory on the M2 on $\mathbb{R}^4 \times (\mathbb{R}^4/\mathbb{Z}_k)$ as the IR fixed point of a field theory with a lagrangian description. This is the $\mathbb{Z}_k$ quiver theory of the 2+1 SYM. From the point of view of branes, this amounts to considering the decoupling limit of M2-branes on $\mathbb{R}^3 \times S_1 \times (\mathbb{R}^4/\mathbb{Z}_k)$. The supergravity solution for such a brane configuration is easy to find. They are simply

$$ds^2 = H^{2/3}(-dt^2 + dx_1^2 + dx_2^2) + H^{1/3}(dy^2 + y^2 d\Omega_2^2 + dz^2 + dr^2 + r^2 ds^2_{\text{Lens}}) \tag{4.1}$$

where ds^2_{Lens} is the metric on the Lens space which is the base of the $\mathbb{R}^4/\mathbb{Z}_k$ viewed as a cone, and

$$H(y,z,r) = 1 + \sum_{n=-\infty}^{\infty} \frac{Q_{M2}}{(r^2 + y^2 + (z - 2\pi n R)^2)^3}\,. \tag{4.2}$$

It is possible to take the decoupling limit of this solution keeping

$$\frac{R}{l_s^2} = \frac{g_s}{l_s} = g^2_{\text{YM2}} = \text{fixed} \tag{4.3}$$

which will give rise to a different supergravity background describing the renormalization group flow of the 2+1 dimensional $\mathbb{Z}_k$ quiver theory flowing to the same superconformal field theory.

What we have here is a pair of supergravity solutions, both of which asymptotes to the same $AdS_4 \times S_7/\mathbb{Z}_k$. It is therefore a holographic realization of two different RG flows which flow to the same conformal field theory in the far IR. This is mirror symmetry. Although the metric on the supergravity side asymptotes to the same thing near the core, the geometry away from the core of the two solutions are clearly different from each other. This illustrates quite explicitly in the AdS/CFT language the basic fact that mirror symmetry is an equivalence only for the far IR of a pair of field theories.

The basic idea behind the embedding of the superconformal field theory into a lagrangian field theory was to compactify one of the dimensions either in the $\mathbb{R}^4$ or the $\mathbb{R}^4/\mathbb{Z}^k$. The freedom to choose between the two was the basis for mirror symmetry. Let us now consider what happens if one compactifies both so that we have $(\mathbb{R}^3 \times S^1) \times$Taub-NUT. Now there are two ways to reduce the same geometry from M-theory to type IIA. Let us for the sake of the argument reduce on the circle in the Taub-NUT. This will give rise to a D2/D6 system with one of the direction transverse to the D2-brane compactified. In the decoupling limit, compactness of the directions transverse to the world volume of the brane is an indication that the underlying gauge theory is a $U(\infty)/\mathbb{Z}$ theory because of the presence of images. According to the argument of [28], it is better to view this as a theory with one extra dimension. From the point of view of the supergravity, the same picture manifests itself in the fact that in the near horizon limit, where the backreaction of the D2-brane dominates, the proper size of the S^1 transverse to the D2 shrinks as one approaches the boundary. At the point where this proper size becomes smaller than the string length, the supergravity description of this geometry become unreliable, and following the argument of [24], one is instructed to go to the T-dual picture, where the D2 becomes a D3.

Unfortunately, the same T-duality maps the D6 to a D5. T-duality in supergravity is not capable of handling this map except for the case where the D5 is completely smeared. We are therefore unable to provide a purely supergravity description of the 3+1 dimensional UV fixed point for the decoupled theory on M2 in $(\mathbb{R}^3 \times S^1) \times$ Taub-NUT. The problem of finding this supergravity solution was attempted most recently in [18]. Let us note in passing that at least the solution for the case of the smeared D5 considered in section 4.1 of [18] can be obtained from (2.16) by applying the T-duality rules for supergravity.

Another interesting aspect of the decoupled theory of M2 on $(\mathbb{R}^3 \times S^1) \times$ Taub-NUT is the fact that its UV description on the field theory side is precisely the defect field theory introduced in [29, 30]. In fact, one of the main motivations of [18] was to find the purely gravitational holographic description of the defect field theory. By reducing from M-theory to IIA on the circle of the Taub-NUT and T-dualizing to IIB on the circle in $\mathbb{R}^3 \times S^1$, we arrive at a defect theory consisting only of the D5 defects. The lagrangian of this theory was worked out in [31]. Alternatively, one could have reduced from M-theory to IIA on the circle in $\mathbb{R}^3 \times S^1$ and T-dualizing along the circle of the Taub-NUT arriving at the theory with NS defects. The relation between the two ways of going from M-theory to type IIB is the natural extension of mirror symmetry. Clearly, from the point of view of the type IIB theory, this equivalence is S-duality. What one learns here is that the origin of mirror symmetry in 2+1 dimensional gauge theory is the S-duality of the defect field theory in 3+1 dimensions.

This idea of embedding a 2+1 dimensional theory in some UV structure to make mirror symmetry manifest is not a new idea. Embedding to string theory was exploited for this goal some time ago in [32, 33]. Embedding of the 2+1 dimensional theory into the 3+1 dimensional defect field theory amounts to taking the decoupling limit of [33]. Although the formulation of defect field theories was strongly motivated by string theory, their existence is independent of string theory. The relation between S-duality of the defect field theory and the mirror symmetry of its dimensionally reduced theory can be studied by exploiting their holographic duality, instead of their embedding, to string theory. The former is physically economical.

By embedding a pair of 2+1 dimensional mirror theories into a pair of S-dual defect field theories, one obtains a mirror duality which applies at all scales, not just in the far IR. So the embedding into the defect field theories can be interpreted as an intricate UV modification of the mirror pair theories so as to extend their range of validity beyond the far IR. For the abelian case, this issue was addressed in [34]. For the non-abelian case, the natural UV modification appears to involve a theory with one extra dimension and defects.

5. Some generalizations

In the previous sections, we discussed mainly the localized D2/D6 supergravity configuration where all of the D2 and D6 are coincident. Let us now consider some generalizations.

5.1 Separating D2 from D6

One simple generalization one can consider is separating the D2-brane from the D6-brane. For the sake of concreteness, let us consider the case where there is one of each of D2 and D6.

From the point of view of M-theory, this is a configuration of a single M2-brane at a generic point on the Taub-NUT background (2.2). Therefore, the supergravity solution is given by the same ansatz (2.1) where H is a solution of the harmonic equation (2.6) but with a source located at a generic point in the Taub-NUT. From the point of view of finding a localized D2/D6 solution of the type IIA supergravity equation of motion, however, one is only interested in sources which are smeared along the 11-th coordinate ψ. Let us therefore take the M2-brane source to be smeared evenly along ψ as this would also simplify the analysis. The harmonic function H is then a solution of

$$(\nabla_{TN}^2 + \nabla_y^2)H = 2\pi^4 Q_{M2}\delta^4(\vec{y}-\vec{y}_0)\delta^3(\vec{r}-\vec{r}_0)\delta(\psi-\psi_0)\,. \tag{5.1}$$

The factor of $2\pi^4$ arises from the fact that

$$\nabla_r^2\left(\frac{1}{r^6}\right) = 2\pi^4\delta^8(r) \tag{5.2}$$

in 8 dimensions. Smearing along ψ and separating the variables as was done in (2.8), one finds that $H_p(\vec{r})$ is a solution of

$$\left(\nabla_r^2 - \frac{2mp^2}{|\vec{r}-\vec{r}_0|} - p^2\right)H_p(\vec{r}) = \frac{2\pi^4 Q_{M2}}{2\pi R}\delta^3(\vec{r}-\vec{r}_0)\,, \qquad R = 4m \tag{5.3}$$

with appropriate boundary conditions. This equation is of the form

$$\left(\nabla_r^2 + \frac{2k\nu}{|\vec{r}-\vec{r}_0|} + k^2\right)G(\vec{r},\vec{r}_0) = \delta^3(\vec{r}-\vec{r}_0) \tag{5.4}$$

if one identifies

$$H_p(\vec{r}) = \frac{2\pi^4}{2\pi R}G(\vec{r},\vec{r}_0)\,, \qquad \nu = imp\,, \qquad k = ip\,. \tag{5.5}$$

Precisely this equation with the appropriate boundary conditions was considered in [35] and the solution was found to be

$$G(\vec{r},\vec{r}_0) = -\frac{\Gamma(1-i\nu)}{4\pi|\vec{r}-\vec{r}_0|}\frac{1}{ik}\left(-\frac{\partial}{\partial y}+\frac{\partial}{\partial x}\right)\mathcal{W}_{i\nu,1/2}(-ikx)\mathcal{M}_{i\nu,1/2}(-iky) \tag{5.6}$$

where $\mathcal{M}_{a,b}(z)$ and $\mathcal{W}_{a,b}(z)$ are the Whittaker functions, and

$$x = r + r_0 + |\vec{r}-\vec{r}_0|\,, \qquad y = r + r_0 - |\vec{r}-\vec{r}_0|\,. \tag{5.7}$$

If one sends the source $\vec{r}_0$ to zero, $G(\vec{r},\vec{r}_0)$ simplifies to

$$G(\vec{r}) = \frac{1}{4\pi r}\Gamma(1-i\nu)\mathcal{W}_{i\nu,1/2}(-2ikr)\,. \tag{5.8}$$

Using the identity

$$\mathcal{W}_{\kappa,\mu} = z^{\mu+1/2}e^{-z/2}\mathcal{U}\left(\frac{1}{2}+\mu-\kappa, 1+2\mu, z\right) \tag{5.9}$$

and the identification (5.5), one can show that (5.8) is equivalent to (2.11) including all the numerical factors.

Using (5.6) and the identification (5.5), one can write an explicit expression for the supergravity solution of the localized D2/D6 configuration. Since harmonic equations are linear, it is straightforward to generalize this to the case where the D2 is distributed arbitrarily in transverse coordinates. It is also straightforward to generalize this solution to the case where there are multiple D6's as long as all of the D6's are coincident. Simply set

$$\nu = iN_6 mp\,. \tag{5.10}$$

By scaling $\vec{r}_0 = \alpha' \vec{U}_0$ keeping $\vec{U}_0$ fixed, one can take the decoupling limit of the D2/D6 as we did in section 3. From the point of view of the field theory dual, this corresponds to turning on a vacuum expectation value of some of the adjoint scalars, so that the matter fields in the fundamental acquire mass.

5.2 Separating the D6

Another possible generalization one might consider is to separate the D6's from one another. The M-theory lift of this configuration is the multi-centered Taub-NUT geometry whose metric is given by

$$ds^2 = V d\vec{r}^2 + \frac{(4m)^2}{V}(d\psi + \vec{\omega}\cdot d\vec{r})^2 \tag{5.11}$$

where

$$V(\vec{r}) = 1 + \sum_{i=1}^{N_6} \frac{2m}{|\vec{r}-\vec{r}_i|}\,, \qquad \nabla V = 4m \nabla \times \vec{\omega}\,. \tag{5.12}$$

Applying the same ansatz as (2.1) gives rise to a new harmonic equation (2.6). Although it is not absolutely necessary to do so, let us smear the M2 source along the ψ coordinates. This will simplify the analysis. Then, the harmonic equation (2.6) can be written explicitly as

$$\left(\nabla_r^2 - p^2 V(\vec{r})\right) H_p(\vec{r}) = \frac{2\pi^4 Q_{M2}}{2\pi R}\delta^3(\vec{r}-\vec{r}_0)\,. \tag{5.13}$$

For a geneneral multi-centered Taub-NUT background, this is still a difficult equation to solve. The case of all the D6 being coincident gives rise to a single centered Taub-NUT considered in the previous section.

It turns out that double centered Taub-NUT also admits natural coordinates in which the harmonic equation (5.13) separates [36]. Let us consider this case in some detail.

Consider a double centered Taub-NUT with N_1 coincident centers at $\vec{r}_1$ and N_2 coincident centers at $\vec{r}_2$. Without loss of generality, we can set $\vec{r}_2 = -\vec{r}_1$. One can then introduce the so called prolate spheroidal coordinates

$$\xi = \frac{|\vec{r}-\vec{r}_1| + |\vec{r}-\vec{r}_2|}{2L}\,, \qquad \eta = \frac{|\vec{r}-\vec{r}_1| - |\vec{r}-\vec{r}_2|}{2L} \tag{5.14}$$

where $2L = |\vec{r}_1 - \vec{r}_2|$ is the distance between the two centers. The ranges of these coordinates are $1 < \xi$ and $-1 < \eta < 1$. The countours of fixed ξ and η are illusterated in figure 2.

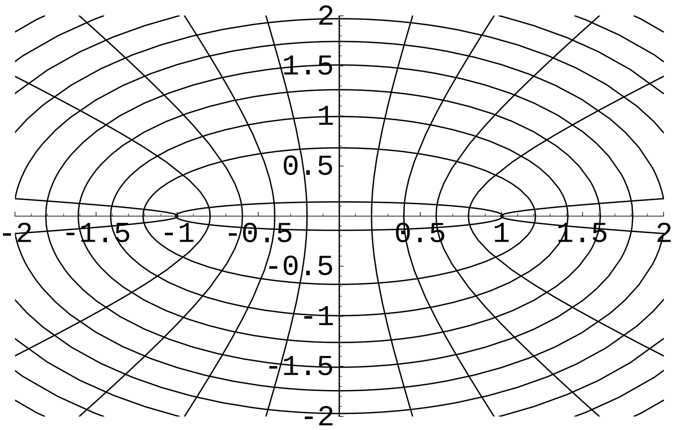

Figure 2: Contours of fixed prolate spheroidal coordinates ξ and η.

Let ϕ denote the angular coordinate around a symmetry axis defined by $\vec{r}_1 - \vec{r}_2$. Then, the set of coordinates ξ, η, and ϕ specifies a point $\vec{r}$. In these coordinates, the harmonic equation (5.13) becomes

$$\left(\partial_\xi(\xi^2-1)\partial_\xi + \partial_\eta(1-\eta^2)\partial_\eta + \left(\frac{1}{\xi^2-1}+\frac{1}{1-\eta^2}\right)\partial_\phi^2 - L^2(\xi^2-\eta^2)Vp^2\right) H_p(\xi,\eta,\phi) =$$
$$= \frac{2\pi^4 Q_{M2}}{2\pi LR}\delta(\xi-\xi_0)\delta(\eta-\eta_0)\delta(\phi-\phi_0) \tag{5.15}$$

where

$$(\xi^2-\eta^2)V = (\xi^2-\eta^2) + \frac{2m(N_1+N_2)\xi}{L} - \frac{2m(N_1-N_2)\eta}{L}\,. \tag{5.16}$$

To further analyze this problem, it is convenient to consider the solution to the equation

$$\left(-\partial_\eta(1-\eta^2)\partial_\eta + \frac{k^2}{1-\eta^2} - p^2L^2\eta^2 - 2p^2mL(N_1-N_2)\eta\right) B_{\lambda,k}(\eta) = \lambda B_{\lambda,k}(\eta)\,. \tag{5.17}$$

This equation takes the form

$$\frac{1}{\sin\theta}\frac{d}{d\theta}\sin\theta\frac{d}{d\theta}B_{\lambda,k} + \left(\lambda - \frac{k^2}{\sin^2\theta} + 2p^2mL(N_1-N_2)\cos\theta + p^2L^2\cos^2\theta\right) B_{\lambda,k} = 0 \tag{5.18}$$

after making the change of variables

$$\eta = \cos\theta\,, \qquad 0 < \theta < \pi\,. \tag{5.19}$$

In the $L \to 0$ limit, this equation becomes the Legendre equation. For finite L, one expects a discrete spectrum of eigenvalues λ_n and its associated eigenfunction $B_{\lambda_n,k}(\eta)$. They can be determined either using numerical methods, perturbation theory, or by expanding

$$B_{\lambda,k}(\eta) = e^{pL\eta}\sum_{s=0}^{\infty} c_s P_{s+k}^k(\eta)\,, \tag{5.20}$$

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and deriving a recursion relation for the coefficients c_s along the lines of [37]. We will consider these eigenfunctions to be orthonormalized so that

$$\int_0^\pi d\theta \sin(\theta)\, B_{\lambda_m,k}(\cos\theta) B_{\lambda_n,k}(\cos\theta) = \int_{-1}^1 d\eta\, B_{\lambda_m,k}(\eta) B_{\lambda_n,k}(\eta) = \delta_{mn} \,. \tag{5.21}$$

One now sees that upon parameterizing

$$H_p(\xi,\eta,\psi) = \sum_{n,k} A_{\lambda_n,k}(\xi) B_{\lambda_n,k}(\eta) e^{ik\phi} \,, \tag{5.22}$$

where $B_{\lambda_n,k}$ are orthonormal, $A_{\lambda_n,k}(\xi)$ satisfies

$$\begin{aligned} &\left(\partial_\xi(\xi^2-1)\partial_\xi - \frac{k^2}{\xi^2-1} - p^2(L^2\xi^2 + 2m(N_1+N_2)L\xi) - \lambda_n\right) A_{\lambda_n,k}(\xi) = \\ &\quad = \frac{2\pi^4 Q_{M2}}{2\pi LR} B_{\lambda_n,k}(\eta_0) e^{-k\phi_0} \delta(\xi-\xi_0) \,. \end{aligned} \tag{5.23}$$

Although somewhat complicated, this is a linear inhomogenious ordinary differential equation which can be solved numerically or using the method of [37]. One can then evaluate

$$H(y,\xi,\eta,\phi) = 1 + \int \frac{d^4p}{(2\pi)^4} e^{ipy} \sum_{n,k} A_{\lambda_n,k}(\xi) B_{\lambda_n,k}(\eta) e^{ik\phi} \tag{5.24}$$

which when substituted into (2.1) gives rise to the supergravity solution of the D2 parallel to two collections of D6-branes.

6. Concluding remarks

The main goal of this paper was the construction of the localized D2/D6 supergravity solution. We have constructed this solution explicitly as an integral expression in (2.15). Using this solution, it was possible to construct a holographic dual to 2+1 dimensional Yang-Mill with matter in the fundamental representation, and describe mirror symmetry in the language of AdS/CFT correspondence.

It would be interesting to explore the standard holographic observables: entropy, Wilson loop, and correlation functions, for this supergravity solution. It would also be interesting to explore the mapping of observables between the mirror pairs from the point of view of holography.

The D2/D6 system appears to be unique in providing a conceptually clean setup to add flavors to AdS/CFT. The D3/D7 system suffers from the lack of asymptotic freedom, and the D1/D5 system suffers from the lack of moduli-space. One can still describe the decoupled theory on D1/D5 in Born-Oppenheimer approximation, but that does not appear to be compatible with the holographic duality, which has as a starting point a stationary supergravity solution with definite configuration of static sources. One manifestation of this difficulty is the fact that a localized supergravity solution of D1 coincident with D5 does not even appear to exist [9].

The key to the simplicity is the fact that the D6-brane lifts to a Taub-NUT geometry in M-theory which is purely geometrical. Furthermore, the geometry is sufficiently regular both near the core and at infinity. This is what made generalization of the D2/D6 intersection in the near core region considered in [5] to the full Taub-NUT geometry possible. It would be very interesting to see if the D4/D6 intersection considered in [6] can be extended in a similar manner by taking advantage of the simplicity of the Taub-NUT geometry.

A different simple generalzation is the supergravity dual of the non-commutative gauge theory with fundamental matter. Such a solution can be found by applying the same T-duality transformation considered in [38] or by following the twist operation for the dipole theories introduced in [39]. The resulting supergravity background is similar to (3.8) but with the metric along the D2-brane worldvolume replaced by

$$\frac{ds^2}{l_s^2} = h(Y,U)^{-1/2}\left(1+\frac{g_{\mathrm{YM2}}^2 N_6}{2U}\right)^{-1/2}\left(-dt^2 + \frac{dx_1^2+dx_2^2}{1+\Delta^4 h(Y,U)^{-1}\left(1+\frac{g_{\mathrm{YM2}}^2 N_6}{2U}\right)^{-1}}\right)+\cdots \tag{6.1}$$

where $2\pi\Delta^2 = \theta^{12}$ is the non-commutativity parameter along the D2-brane world volume.

There are other generalizations one might consider. For example, one can separate the D6's completely, so that the M-theory background becomes that of a multi-center Taub-NUT. One might also consider finding the supergravity solution for a decoupled theory on M2 probing a manifold of Sp(2) holonomy[2] considered in [3]. For these cases, separation of variables do not appear to work and more sophisticated methods for solving for the the harmonic function must to be employed.

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